

# Complex Symmetry of Composition Operators

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## Resumo

A bounded operator  $T$  on a complex separable Hilbert space  $\mathcal{H}$  is said to be *complex symmetric* if there exists an orthonormal basis for  $\mathcal{H}$  with respect to which  $T$  has a self-transpose matrix representation. An equivalent way to define complex symmetry is the following: if a *conjugation* is a conjugate-linear operator  $C : \mathcal{H} \rightarrow \mathcal{H}$  that satisfies the conditions

- (a)  $C$  is isometric:  $\langle Cf, Cg \rangle = \langle g, f \rangle \forall f, g \in \mathcal{H}$ ,
- (b)  $C$  is involutive:  $C^2 = I$ ,

then we say that  $T$  is complex symmetric if there exists a conjugation  $C$  such that  $T = CT^*C$ . Suppose  $H^2(\mathbb{B}_n)$  is the classical Hardy space of analytic functions on the unit ball  $\mathbb{B}_n \subset \mathbb{C}^n$  and define the composition operator  $C_\psi$  on  $H^2(\mathbb{B}_n)$  by  $C_\psi f = f \circ \psi$ , where  $\psi$  is an analytic self-map of  $\mathbb{B}_n$ . In this presentation, a solution is given to problem posed by Stephan Ramon Garcia and Christopher Hammond [1]: *If  $\varphi$  is an involutive Moebius automorphism of  $\mathbb{B}_n$ , find a conjugation operator  $\mathcal{J}$  on  $H^2(\mathbb{B}_n)$  such that  $C_\varphi = \mathcal{J}C_\varphi^*\mathcal{J}$ .*

## Referências

- [1] S. R. Garcia and C. Hammond, *Which weighted composition operators are complex symmetric?* Operator Theory: Advances and Applications 236 (2014), 171-179.
- [2] S. Waleed Noor, *Complex symmetry of composition operators induced by involutive ball automorphisms*, Proc. Amer. Math. Soc. 142 (2014), no. 9, 3103-3107.